BARRIER COOLING WITH MULTISLIT AND LATTICE INJECTION IN A TURBULENT BOUNDARY LAYER

E. P. Volchkov, S. S. Kutateladze, V. Ya. Levchenko, and A. I. Leont'ev

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 149–153, 1966

An analytical method is proposed for calculating the effectiveness of barrier cooling of a flat, heat-insulated wall with multislit and lattice supply of the cooling gas. The dimensionless temperature of the heat-insulated wall is taken as the measure of the effectiveness of heat protection.

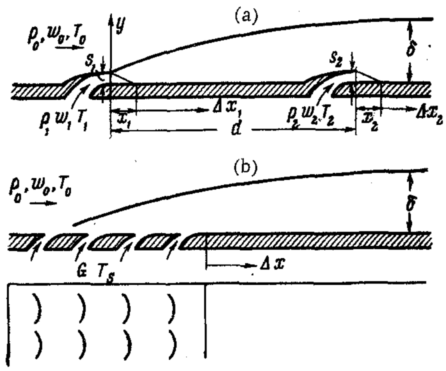

Fig. 1. Diagrams of a) multislit and b) lattice cooling.

At first glance, the flow patterns in the boundary layer appear to be more complicated for the cases under consideration than when the cooling gas is blown in through a single slit. However, it seems possible to extend the methods of calculation proposed in [1, 2] to these cases.

Let us consider a homogeneous turbulent boundary layer of gas with constant physical properties in the given temperature range. Gas with temperature T_0 [°K] and velocity w_0 [m/sec] flows over the surface (Fig. 1a). The cooling gas is injected through a series of successive slits with width $s_1, \ldots, s_n$ [m] with temperatures $T_1, \ldots, T_n$ and velocities $w_1, \ldots, w_n$, respectively. Immediately behind the section of each slit is a zone $x_1, \ldots, x_n$ in which the wall temperature does not vary and is equal to the temperature of the injected gas. The extent of this zone can be found to the first approximation by making use of known formulas for submerged jets [3].

There is no heat exchange through the wall and its temperature is a function of the x coordinate. The wall temperature behind the first slit can be found by the calculations for single-slit cooling, for example, as in [1, 2]. The problem consists in determining T_w^* behind the second slit, the third slit, and so on. Then it will be necessary to determine the characteristic parameters of the boundary layer in the section of each slit.

1. The energy thickness in the section of the second slit can be expressed in the following manner:

$$\delta_{T2}^{**} = \int_0^\infty \frac{\rho w}{\rho_0 w_0}\left(\frac{T_0 - T}{T_0 - T_2}\right) dy =$$

$$= \int_0^{s_2} \frac{\rho w}{\rho_0 w_0}\left(\frac{T_0 - T}{T_0 - T_2}\right) dy + \int_{s_2}^\infty \frac{\rho w}{\rho_0 w_0}\left(\frac{T_0 - T}{T_0 - T_2}\right) dy =$$

$$= m_2 s_2 + \frac{T_0 - T_{w2}^*}{T_0 - T_2}(\delta_{T_0}^{**})_2 \qquad \left(m_2 = \frac{\rho_2 w_2}{\rho_0 w_0}\right), \tag{1.1}$$

where m_2 is the injection parameter of the second slit, T_{W2}^* the wall temperature above the second slit.

The integral

$$(\delta_{T_0}^{**})_2 = \int_{s_2}^\infty \frac{\rho w}{\rho_0 w_0}\left(\frac{T_0 - T}{T_0 - T_{w2}^*}\right) dy$$

is the energy thickness of the boundary layer above the slit in the section of the second slit. Integrating the equation for the energy of the boundary layer from x = 0 (the section of the first slit) to x = d (the section of the second slit); with $q_W = 0$,

$$\frac{d\delta_T^{**}}{dx} + \frac{\delta_T^{**}}{\Delta T}\frac{d(\Delta T)}{dx} = 0, \tag{1.2}$$

we find that the effectiveness of the protection of the wall above the second slit is equal to

$$\frac{T_0 - T_{w2}^*}{T_0 - T_1} = \frac{\delta_{T1}^{**}}{(\delta_{T_0}^{**})_2}$$

$$(\delta_{T_1}^{**} = m_1 s_1). \tag{1.3}$$

Here T_1^{**} is the energy thickness in the section of the second slit [1, 2]. Correspondingly, it follows from equalities (1.1) and (1.4) that

$$\delta_{T2}^{**} = m_2 s_2 + \frac{T_0 - T_1}{T_0 - T_2}\delta_{T1}^{**} = m_2 s_2 + \frac{T_0 - T_1}{T_0 - T_2} m_1 s_1. \tag{1.4}$$

It can be shown in a similar manner that the energy thickness in the section of the third slit is expressed in the following manner (in this case, the equation for the energy of the boundary layer (1.2) is integrated on the section between the second and third slits):

$$\delta_{T3}^{**} = m_3 s_3 + \frac{T_0 - T_2}{T_0 - T_3}\delta_{T2}^{**} =$$

$$= m_3 s_3 + \frac{T_0 - T_2}{T_0 - T_3} m_2 s_2 + \frac{T_0 - T_1}{T_0 - T_3} m_1 s_1. \tag{1.5}$$

And finally, in the section of the n-th slit

$$\delta_{Tn}^{**} = m_n s_n + \frac{T_0 - T_{n-1}}{T_0 - T_n}\delta_{T,n-1}^{**}, \tag{1.6}$$

or

$$\delta_{Tn}^{**} = m_n s_n + \frac{T_0 - T_{n-1}}{T_0 - T_n} m_{n-1} s_{n-1} +$$

$$+ \frac{T_0 - T_{n-2}}{T_0 - T_n} m_{n-2} s_{n-2} + \ldots + \frac{T_0 - T_1}{T_0 - T_n} m_1 s_1. \tag{1.7}$$

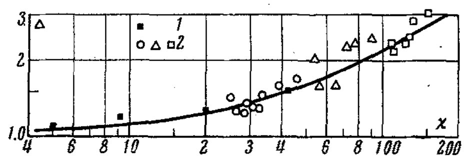

Fig. 2. Changes in momentum thickness with injection through a tangential slit ($w_s/w_0 \leq 1$); points: 1) experimental [4]; 2) experimental [5].

2. The momentum thickness is determined from solving the equation for momenta of the boundary layer, which is of the following form for zero-gradient flow:

$$\frac{dR^{**}}{dR_x} = \frac{C_f}{2} \qquad \left(R^{**} = \frac{\rho_0 w_0 \delta^{**}}{\mu_0},\ R_x = \right.$$

$$\left. = \frac{\rho w_0 x}{\mu_0},\ \delta^{**} = \int_0^\infty \frac{\rho w}{\rho_0 w_0}\left(1 - \frac{w}{w_0}\right) dy\right). \tag{2.1}$$

Here, R^{**} is the Reynolds number constructed for the momentum thickness; R_x is the Reynolds number constructed for the longitudinal coordinate; C_f is the local value of the friction coefficient; δ^{**} is

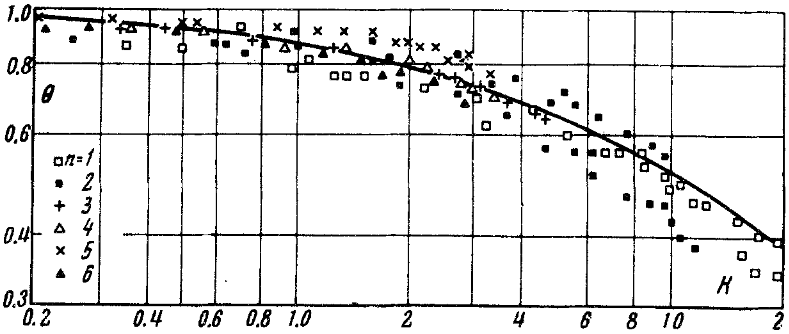

Fig. 3. The curve for formula (3.6); points: experiments [8] with $w_s/w_0 < 0.2$; $0 < d/s < 78$.

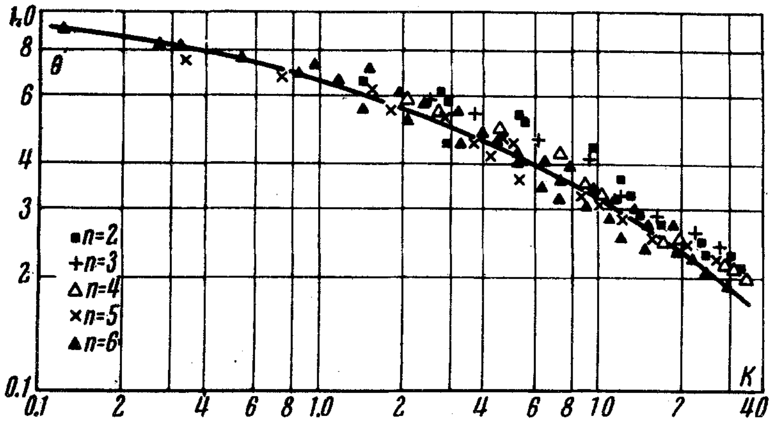

Fig. 4. Curve for formula (3.7); points: experiments [8] with $0.615 < w_s/w_0 < 1.33$; $0 < d/S < 78$.

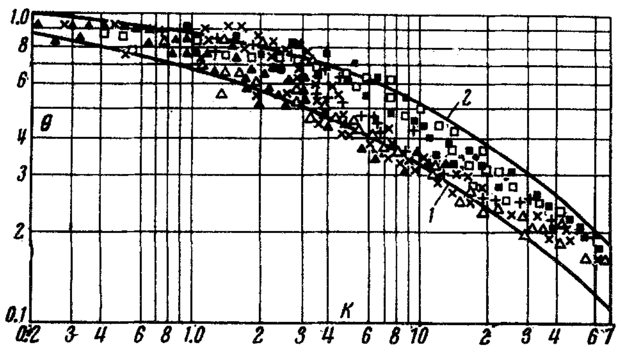

Fig. 5. Generalization of data on cooling a heat-insulated wall with multislit injection; curves 1 and 2) from formulas (3.6) and (3.7). The experimental points [8] with $0 < w_s/w_0 < 1.33$ have the same notation as in Figs. 3 and 4.

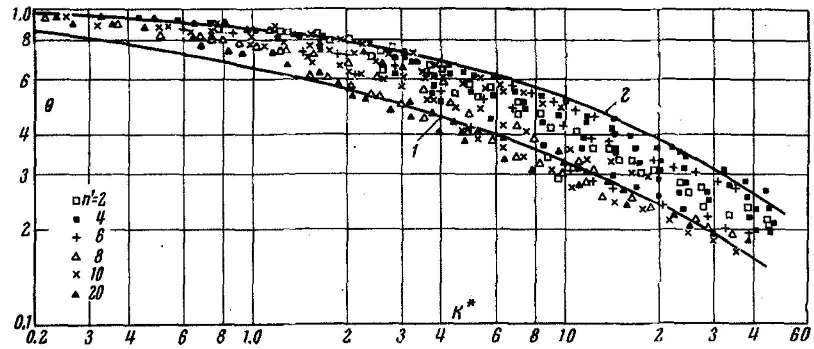

Fig. 6. Effectiveness of thermal protection Θ versus K* with the cooling gas injected through a tangential lattice with $0 < w_s/w_0 < 1$; curves 1 and 2) calculations from formulas (3.6) and (3.7); points: experiments [8]; in this case, H' is the number of open rows of slits.

the momentum thickness. We consider, as in [1, 2], that the boundary layer on the wall is turbulent with a power-law velocity profile. In this case, the law of friction is valid in the form

$$\tfrac{1}{2}C^f = AR^{**-a}. \tag{2.2}$$

Integrating Eq. (2.1) with (2.2) taken into consideration, on the section between the first and second slits, we obtain

$$R^{**} = [R_1^{**(a+1)} + A(a+1)R_x]^{\frac{1}{a+1}}. \tag{2.3}$$

For the power-law velocity profile with exponent n = 1/7, the computations yield the following values: A = 0.0128 and $a = 0.25$. After transformations of (2.3), we have

$$\Delta = [1 + 0.016\chi^{1.25}]^{0.8} \qquad \left(\Delta = \frac{\delta^{**}}{\delta_1^{**}}, \quad \chi = \frac{x}{\delta_1^{**}R_x^{0.2}}\right). \tag{2.4}$$

Here δ_1^{**} is the momentum thickness in the section of the first slit. The relationship $\Delta = \Delta(\chi)$ constructed in Fig. 2 from formula (2.4) is satisfactorily substantiated by the results of experiments [4, 5]. The momentum thickness of the boundary layer above the slit in the section of the second slit is found from (2.4),

$$(\delta_0^{**})_2 = \left[\delta_1^{**1.25} + 0.016\left(\frac{d}{R_d^{0.2}}\right)^{1.25}\right]^{0.8}. \tag{2.5}$$

The total momentum thickness in this section, taking injection through the slit into consideration, is

$$\delta_2^{**} = m_2 s_2\left(1 - \frac{w_2}{w_0}\right) + (\delta_0^{**})_2 . \tag{2.6}$$

The momentum thickness in the section of the n-th slit can be found in a like manner,

$$\delta_n^{**} = m_n s_n\left(1 - \frac{w_n}{w_0}\right) + (\delta_0^{**})_n . \tag{2.7}$$

Here $(\delta_0^{**})_n$ is found by successive integration of the momentum equation in form (2.1), taking the law of friction (2.2) into account in the sections between the slits.

The local friction coefficient behind the n-th slit is found from Eqs. (2.1) and (2.2) with the boundary-value condition (2.7),

$$\frac{C_f}{2} = \frac{0.0128}{[R_n^{**1.25} + 0.016R_{xn}]^{0.2}}. \tag{2.8}$$

Here x_n is the distance measured from the section of the n-th slit.

3. Let us consider the effectiveness of thermal protection. It can be seen from Fig. 2 that the local value of the momentum thickness differs considerably from the value in the section of the slit only at great distances.

It was shown in [6, 7] that local changes in the dynamic flow field have only a secondary effect on the process of heat transfer. In this connection, when analyzing the effectiveness of thermal protection, one may assume to the first approximation that the change in momentum to the n-th slit takes place only through injection through the slit (that is, we neglect friction on the wall between slits to the n-th slit).

Then the expression for the momentum thickness in the section of the n-th slit is simplified and it can be represented in the form

$$\delta_n^{**} = \int_0^\infty \frac{\rho_1 w_1}{\rho_0 w_0}\left(1 - \frac{w_1}{w_0}\right)dy + \int_0^\infty \frac{\rho_2 w_2}{\rho_0 w_0}\left(1 - \frac{w_2}{w_0}\right)dy + \ldots$$
$$\ldots + \int_0^\infty \frac{\rho_n w_n}{\rho_0 w_0}\left(1 - \frac{w_n}{w_0}\right)dy = m_1 s_1\left(1 - \frac{w_1}{w_0}\right) +$$
$$+ m_2 s_2\left(1 - \frac{w_2}{w_0}\right) + \ldots + m_n s_n\left(1 - \frac{w_n}{w_0}\right). \tag{3.1}$$

The formulas for calculating the effectiveness of the thermal protection of the heat insulated wall when the cooling gas is injected through a series of successive slits can be obtained by making use of methods proposed for the case when the cooling gas is injected through a single slit [1, 2]. The difference consists only in determining the initial parameters of the boundary layer which are calculated in the section of the n-th slit, taking the injection of the cooling gas through all the preceding slits into consideration by (1.7) and (3.1). These quantities are taken into consideration as boundary conditions when integrating the equations of energy (1.2) and momenta (2.1) of the boundary layer on the wall behind the n-th slit.

Making use of the procedures of [2], it can be shown that the formulas derived there for the effectiveness of the heat shield can also be extended to the case of multislit injection of the cooling gas.

Following [2], for the power velocity profile with n = 1/7, we obtain

$$\Theta = \left[\left(\frac{R_{Tx}^{**}}{R_{T\Delta x}^{**}}\right)^{0.25}\left(\frac{R_{\Delta x}^{**}}{R_x^{**}}\right)^{0.107} - 1\right]^{0.8}\left(\frac{R_{Tn}^{**}}{R_{Tx}^{**}}\right)^{0.2},$$
$$\Theta = \frac{T_0 - T_w^*}{T_0 - T_n}, \quad \begin{aligned} R_{Tx}^{**} &= [R_{Tn}^{**1.25} + 0.016\,R_{\Delta xn}]^{0.8}, \\ R_x^{**} &= [R_n^{**1.25} + 0.016\,R_{\Delta xn}]^{0.8}, \\ R_{T\Delta x}^{**} &= R_{\Delta x}^{**} = [0.016\,R_{\Delta xn}]^{0.8}. \end{aligned} \tag{3.2}$$

Henceforth, we shall assume, for the sake of simplicity, that

$$w_s = w_1 = \ldots = w_n, \; T_s = T_1 = \ldots = T_n, \; s_1 = \ldots = s_n. \tag{3.3}$$

Then, we have from (1.7) and (3.1)

$$\delta_{Tn}^{**} = nms, \qquad \delta_n^{**} = nms\left(1 - \frac{w_s}{w_0}\right). \tag{3.4}$$

Under these conditions, (3.2) can be transformed to the following form:

$$\Theta = \frac{1}{(1 + 0.016\,K)^{0.16}} \Big\{(1 + 62.5\,K^{-1})^{0.2} \Big[1 + $$
$$+ 62.5 \left(1 - \frac{w_s}{w_0}\right)^{1.25} K^{-1}\Big]^{-0.036} - 1\Big\}^{0.8},$$

$$\Theta = \frac{T - T_w^*}{T_0 - T_s}, \qquad K = \frac{R_{\Delta xn}}{R_{Tn}^{**1.25}} = \frac{R_{\Delta xn}}{R_{ns}^{1.25}}, \quad R_{ns} = \frac{\rho_s w_s ns}{\mu_0}. \quad (3.5)$$

From expression (3.5) we obtain the following interpolation formulas for the limiting cases:

$$\Theta = \left[\left(1 + \frac{62.5}{K + 0.143}\right)^{0.111} - 1\right]^{0.8} (1 +$$
$$+ 0.016K)^{-0.16} \quad \text{for} \quad \frac{w_s}{w_0} \ll 1, \qquad (3.6)$$

$$\Theta = \left[\left(1 + \frac{62.5}{K + 2}\right)^{0.2} - 1\right]^{0.8} (1 + 0.016\,K)^{-0.16} \quad \text{for} \quad \frac{w_s}{w_0} \approx 1. \quad (3.7)$$

A comparison of the relationship $\Theta = \Theta(K)$ constructed from formulas (3.6) and (3.7) is given in Figs. 3-5.

In experiments [9] involving multislit and lattice injection of the cooling gas, there was a certain dynamic and thermal boundary in the main flow upstream from the first slit (that is, there was an initial energy thickness caused by cooling of the gas in the boundary layer before the working section). Thus, in comparison with calculations, we considered only those experiments in which the energy thickness due to injection through slits would far exceed the initial energy thickness due to cooling of the main flow through the wall up to the first slit (that is, experiments in which nms > 1 mm).

It can be seen from Figs. 3-5 that the calculations are confirmed by experimental data on multislit cooling of a heat-insulated wall.

Experimental data is given in [8] on injection of the cooling gas through a lattice panel in which tangential slits were cut (Fig. 1c).

In our case, the parameter K on formulas (3.5)-(3.7) lead to the following formula, which is convenient for practical use:

$$K = (\mu_0/G)^{1.25}\, R\Delta x.$$

Here, G is the coolant consumption per unit width of surface.

When expressed in this way, there is no problem of determining slits of equivalent size, as was done in reference [8]. As can be seen from Fig. 6, even in such a case, which is complicated at first glance, the calculations are substantiated by experimental data [8].

REFERENCES

1. Collection: Friction and Heat Transfer in the Boundary Layer [in Russian], ed. S. S. Kutateladze, Izd-vo SO AN SSSR, 1964.

2. E. P. Volchkov and V. Ya. Levchenko, "Effectiveness of gas film cooling for a turbulent boundary layer," PMTF [Journal of Applied Mechanics and Technical Physics], no. 5, 1965.

3. G. N. Abramovich, Theory of Turbulent Jets [in Russian], Fizmatgiz, 1960.

4. Hartnett, Eckert, and Birkebak, "An analysis of the basic characteristics of a turbulent boundary layer with air delivered through tangential slits," Trans. ASME, Heat Transfer, Series C, vol. 83, no. 3, 1961.

5. R. A. Seban and L. H. Back, "Velocity and temperature profiles in a turbulent boundary layer with tangential injection," Trans. ASME, Heat Transfer, Series C, vol. 84, no. 1, 1962.

6. E. R. G. Eckert and R. M. Drake, Heat and Mass Transfer [Russian translation], Gosenergoizdat, 1961.

7. S. S. Kutateladze and A. I. Leontev, The Turbulent Boundary Layer of a Compressible Gas [in Russian], Izd-vo SO AN SSSR, 1962.

8. Chin, Skirvin, and Bargraf, "Film cooling with multislit and lattice cooling," Trans. ASME, Series C, vol. 83, no. 3, 1961.

21 April 1965 Novosibirsk